

Middle Grades Mathematics (203)

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NES NES Profile: Middle Grades Mathematics (203)

Overview

The resources below provide information about this test, including the approximate percentage of the total test score derived from each content domain. The complete set of the content domains, the test framework, is provided here and contains all of the competencies and descriptive statements that define the content of the test.

Select any of the content domains presented in the chart or its key to view:

- » the test competencies associated with each content domain,
- » a set of descriptive statements that further explain each competency,
- » a sample test question aligned to each competency.

Test Field	Middle Grades Mathematics (203)	
Test Format	t Multiple-choice questions	
Number of Questions	Approximately 150	
Test Duration	Up to 5 hours	
Reference Materials	An on-screen scientific calculator is provided with your test. A formulas page is provided with your test. Reference materials are provided on-screen as part of your test.	



Key	Approximate Percentage of Test	Content Domain	Range of Competencies
	17%	I. Number Sense and Operations	0001–0002
	33%	II. Algebra and Functions	0003–0006
	25%	III. Measurement and Geometry	0007–0009
	25%	IV. Statistics, Probability, and Discrete Mathematics	0010-0012

MIDDLE GRADE MATHEMATICS FORMULAS AND NOTATION

Formula	Description
$V = \frac{1}{3}Bh$	Volume of a right cone and a pyramid
$A=4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$A = \pi r \sqrt{r^2 + h^2}$	Lateral surface area of a right circular cone
$S_n = \frac{n}{2}[2a + (n-1)d] = n\left(\frac{a+a_n}{2}\right)$	Sum of an arithmetic series
$S_n = \frac{a(1 - r^n)}{1 - r}$	Sum of a geometric series
$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \ r < 1$	Sum of an infinite geometric series
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Law of sines
$c^2 = a^2 + b^2 - 2ab \cos C$	Law of cosines
$s^{2} = \frac{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}{n - 1}$	Variance
$s = r\theta$	Arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
\overline{A} is the complement of set A	Set theory
$_{n}P_{r} = P(n,r) = \frac{n!}{(n-r)!}$	
$_{n}C_{r} = C(n,r) = \frac{n!}{(n-r)!r!}$	

Calculator Information

A scientific calculator will be provided with your test. You may not use your own scientific calculator or calculator manual.

Content Domain I: Number Sense and Operations

Competencies:

0001 Understand numbers.

Descriptive Statements:

- » Analyze the relationships between the subsets of the real numbers.
- » Analyze the role of place value in any number system.
- » Analyze the use of estimation in a variety of situations.
- » Translate between different representations of numbers.
- » Apply number-theory concepts (e.g., divisibility rules, prime factorization, greatest common factors) in problem-solving situations.

Sample Item:

In the base-2 number system, the sum of 101 and 1011 is:

- A. 1000
- B. 1112
- C. 2000
- D. 10000

Correct Response and Explanation

D. This question requires the examinee to analyze the role of place value in any number system. Align addends 101 and 1011 such that place values are in the appropriate columns. Add the two digits in the right column, 1 + 1 = 2. The 2 is regrouped as equaling $1 \times 2^1 + 0 \times 2^0$ with the 1 carried to the next column (Table 1). Continue the process until complete (Table 2), regrouping as needed, yielding the sum 10000.

Table	1
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			1	
24	2 ³	2²	21	20
	1	0	1	1
+		1	0	1
				0

1	1	1	1	
24	2 ³	2²	21	20
	1	0	1	1
+		1	0	1
1	0	0	0	0

Table 2

0002 Understand operations.

Descriptive Statements:

- » Analyze relational and operational properties.
- » Analyze a variety of conventional and alternative algorithms.
- » Solve a variety of problems involving integers, fractions, and decimals.
- » Solve a variety of problems involving ratios, proportions, and percents.

Sample Item:

A car that is advertised for \$28,900 is sold after an 8% discount. If the commission rate is 0.75% of the sale price, how much money will the salesperson earn?

- A. \$199.41
- B. \$216.75
- C. \$19,941.00
- D. \$21,675.00

Correct Response and Explanation

A. This question requires the examinee to solve a variety of problems involving ratios, proportions, and percents. The final sale price of the car is 92% of the asking price (100% - 8% discount) and the salesperson earns 0.75% of the sale price. Commission = (\$28,900)(0.92)(0.0075) = \$199.41.

Content Domain II: Algebra and Functions

Competencies:

0003 Understand patterns, relations, and functions.

Descriptive Statements:

- » Analyze a variety of patterns.
- » Analyze the properties of relations and functions in multiple representations (e.g., tables, graphs, equations, words).
- » Analyze direct and inverse proportional relationships.
- » Determine the effects of transformations on the graph of a function or relation.

Sample Item:

A hiker climbs uphill at a steady pace, rests at a scenic spot for a while, then continues at a slower pace to the top of the hill. The hiker stops for lunch at the top, then decides to run down to the base of the hill. Which of the following graphs best expresses the hiker's speed as a function of time?







C. This question requires the examinee to analyze the properties of relations and functions in multiple representations (e.g., tables, graphs, equations, words). The "steady pace" held by the hiker reflects a constant speed over time; thus the first section of the graph is a horizontal line (slope equals zero) at some constant value of speed. The hiker slows to a resting position with a speed of zero, though time passes so the value for time continues to increase. The approach to the top of the hill shows a positive slope, with increasing speed and time as the hiker resumes walking, but since the pace is slower, the graph does not reach the previous level of speed. This pattern repeats when the hiker stops for lunch, but the run to the base reflects a greater speed than was achieved on the initial climb.

0004 Understand algebraic techniques and applications.

Descriptive Statements:

- » Manipulate algebraic expressions, equations, and inequalities (e.g., simplify, transform, factor).
- » Solve linear and nonlinear equations and inequalities.
- » Connect appropriate algebraic notation to phrases and sentences.

Sample Item:

If a person can buy up to 3 times as many desktop computers as laptop computers with the same amount of money, then which of the following inequalities relates the price of a desktop computer D to the price of a laptop computer L?

- A. D ≤ 3L
- B. D≥3L
- C. 3D ≤ L
- D. 3D≥L

Correct Response and Explanation

D. This question requires the examinee to connect appropriate algebraic notation to phrases and sentences. The condition of "up to 3 times as many" is the same as 1 laptop computer having the value of at most 3 desktop computers. Thus for the price of a desktop computer, D, and the price of a laptop computer, L, $3D \ge L$.

0005 Understand linear relations and applications.

Descriptive Statements:

- Analyze the relationship between a linear equation or inequality and its representations.
- Solve systems of linear inequalities or equations with a variety of methods.
- Interpret the meaning of the slope and the y-intercept in various contexts.
- Analyze a variety of real-world problems involving linear equations, systems, and inequalities.

Sample Item:

Which of the following is an equation of a line perpendicular to the line 4x - 9y = 12?

A.
$$y = \frac{9}{4}x + 6$$

B.
$$y = \frac{1}{9}x + 3$$

C.
$$y = -\frac{4}{9}x - 4$$

D.
$$y = -\frac{9}{4}x - 2$$

Correct Response and Explanation

D. This guestion requires the examinee to analyze the relationship between a linear equation or inequality

and its representations. The slope, m_1 , of the given line is found by isolating y in the given equation. $4x - 9y = 12 \implies 9y = 4x - 12 \implies y = \frac{4}{9}x - \frac{12}{9}$ and $m_1 = \frac{4}{9}$. The slope, m_{\perp} , of a line perpendicular to the given line is the negative reciprocal of the slope of the given line m_1 . Thus $m_{\perp} = -\frac{1}{m_1} \text{ so } m_{\perp} = -\frac{9}{4} \text{ and } y = -\frac{9}{4}x - 2$ must be the equation of the perpendicular line.

0006 Understand nonlinear relations and concepts of calculus.

Descriptive Statements:

- » Analyze relationships between multiple representations of a nonlinear equation or inequality.
- » Solve a variety of real-world problems involving nonlinear equations and inequalities.
- » Analyze function behavior in terms of limits, continuity, and rates of change.
- » Apply concepts of calculus to solve problems in real-world situations.

Sample Item:

If the limit of function h(x) equals a real number k as x goes to negative infinity, which of the following is the equation of an asymptote of the graph of h(x)?

- A. $\chi = -k$
- B. y = -k
- C. x = k
- D. y = k

Correct Response and Explanation

D. This question requires the examinee to analyze function behavior in terms of limits, continuity, and rates of change. The situation described can be represented as $\lim_{x \to -\infty} h(x) = k$. This means that as *x* approaches negative infinity, h(x) approaches but never reaches *k* and that y = k is a horizontal asymptote.

Content Domain III: Measurement and Geometry

Competencies:

0007 Understand measurement principles, procedures, and applications.

Descriptive Statements:

- » Analyze the use of various units and unit conversions within the customary and metric systems.
- » Calculate or estimate measures of lengths, areas, and volumes.
- » Apply the concepts of similarity, scale factors, and proportional reasoning to solve indirect measurement problems.
- » Analyze precision, accuracy, and rounding in measurements and computed quantities.

Sample Item:

A car is traveling at a speed of 100 kilometers per hour. What is its approximate speed in meters per second?

- A. 28 meters per second
- B. 36 meters per second
- C. 280 meters per second
- D. 360 meters per second

Correct Response and Explanation

A. This question requires the examinee to analyze the use of various units and unit conversions within the customary and metric systems. Both units in the given quantity must be converted: kilometers to meters and hours to seconds.

 $\frac{100 \text{ kilometers}}{1 \text{ hour}} \cdot \frac{1000 \text{ meters}}{1 \text{ kilometer}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} = 27.8 \frac{\text{meters}}{\text{second}} \approx 28 \frac{\text{meters}}{\text{second}}$

0008 Understand Euclidean geometry in two and three dimensions.

Descriptive Statements:

- » Analyze properties of points, lines, planes, and angles.
- » Use the properties of triangles, quadrilaterals, and other polygons and circles to solve problems.
- » Apply principles of similarity and congruence.
- » Apply the Pythagorean theorem and its converse.
- » Use nets, cross sections, and projections to analyze three-dimensional figures.
- » Analyze geometric arguments using deductive reasoning.

Sample Item:



Five lines intersect as shown. If x and y are angle measures in degrees, which of the following equations relates y to x in the diagram above?

A.
$$y = \frac{1}{3}x$$

B. $y = x - 90$
C. $y = \frac{1}{2}x$
D. $y = 180 - x$

Correct Response and Explanation

B. This question requires the examinee to analyze properties of points, lines, planes, and angles. If two lines are both perpendicular to the same transversal, the two lines are parallel. Thus there are two pairs of parallel lines in the figure. By the alternate interior angle theorem, $x = y + 90^{\circ}$ and $y = x - 90^{\circ}$.

0009 Understand coordinate and transformational geometry.

Descriptive Statements:

- » Analyze two- and three-dimensional figures using coordinate systems.
- Connect algebra and geometry by applying concepts of distance, midpoint, and slope to classify figures and solve problems in the coordinate plane.
- » Analyze transformations of figures in the coordinate plane.
- » Analyze figures in terms of symmetry, and tessellations of the plane.

Sample Item:

The end points of one diagonal of a parallelogram are (1, 3) and (1, -3). The end points of its other diagonal are (3, 1) and (-1, -1). What is the perimeter of the parallelogram?

A.
$$2\sqrt{2} + 2\sqrt{5}$$

- B. 10
- C. 20
- D. $4\sqrt{2} + 4\sqrt{5}$

Correct Response and Explanation

D. This question requires the examinee to connect algebra and geometry by applying concepts of distance, midpoint, and slope to classify figures and solve problems in the coordinate plane. Use points (1, 3) and (3, 1), then points (3, 1) and (1, -3), and the distance formula to find the length of two different sides of the parallelogram: $\sqrt{(1-3)^2 + (3-1)^2} = 2\sqrt{2}$ and $\sqrt{(3-1)^2 + (1-(-3))^2} = 2\sqrt{5}$. There are two of each of these sides, thus the perimeter of the parallelogram is $2 \cdot 2\sqrt{2} + 2 \cdot 2\sqrt{5} = 4\sqrt{2} + 4\sqrt{5}$.

Content Domain IV: Statistics, Probability, and Discrete Mathematics

Competencies:

0010 Understand principles and techniques of statistics.

Descriptive Statements:

- » Analyze the effects of bias and sampling techniques.
- » Use appropriate formats for organizing and displaying data.
- » Analyze univariate and bivariate data in a variety of representations.
- » Make predictions from data presented in a variety of representations.
- » Analyze the use of measures of central tendency and spread.

Sample Item:

Which of the following types of data representation would be most useful for illustrating a correlation between two variables?

- A. a scatter plot
- B. a double bar graph
- C. a histogram
- D. a box-and-whisker plot

Correct Response and Explanation

A. This question requires the examinee to use appropriate formats for organizing and displaying data. The scatter plot is the only one of the four choices that displays data that occur as ordered pairs for two variables.

0011 Understand principles of probability and techniques for determining probability.

Descriptive Statements:

- » Determine probabilities of simple and compound events.
- » Use counting principles to calculate probabilities.
- » Use a variety of visual representations to calculate probabilities.
- » Demonstrate knowledge of methods for simulating probabilities.

Sample Item:

Milkshakes come in chocolate, vanilla, and strawberry flavors. Two people ordered milkshakes, but their orders were lost. If each person is given a randomly chosen flavor, what is the probability that they will both get the flavor that they ordered?

A. 3 B. 9 C. 1 D. 9

Correct Response and Explanation

D. This question requires the examinee to determine probabilities of simple and compound events. The probability of one person getting the right flavor of shake is $\frac{1}{3}$. The probability that the second person gets the right flavor is also $\frac{1}{3}$. Since the two events are independent, the probability of the two people each getting the right flavor shake is $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$.

0012 Understand principles of discrete mathematics.

Descriptive Statements:

- » Apply concepts of permutations and combinations to solve problems.
- » Analyze sequences and series, including limits and recursive definitions.
- » Use finite graphs and trees to represent problem situations.
- » Apply set theory to solve problems.
- Apply principles of logic to solve problems (e.g., conditional and biconditional statements, conjunctions, negations).

Sample Item:

Out of 600 businesses surveyed, 300 had Internet access, 450 had fax machines, and 50 had neither. How many of the businesses surveyed had both Internet access and fax machines?

- A. 100
- B. 150
- C. 200
- D. 250

Correct Response and Explanation

C. This question requires the examinee to apply set theory to solve problems. Finding the number of businesses with both Internet access and fax machines means finding the number of elements in the intersection of the sets represented by I and F as defined below.

Let *I* = the number of businesses with just Internet access. Let *F* = the number of businesses with just fax machines. Let $I \cap F$ = the number of businesses with both Internet access and fax machines.

Equation 1: $F + I + I \cap F = 550$ (600 minus the 50 with neither) Equation 2: $I + I \cap F = 300$ Equation 3: $F + I \cap F = 450$

Substituting $I + I \cap F$ from Equation 2 into Equation 1 gives F + 300 = 550, which implies F = 250.

Substituting this value for *F* in Equation 3 gives $250 + I \cap F = 450$, which implies that $I \cap F = 200$.

